Location-Allocation Problems With The Presence of Barriers

M. D. H. Gamal
Department of Mathematics, FMIPA, University of Riau, Pekanbaru 28293, Indonesia
mdhgamal@unri.ac.id

Abstract. A constructive heuristic is introduced to solve a continuous location-allocation problem in the presence of convex polygonal barriers. Potensial sites for the facility can be anywhere in the space except in the interior of the barriers. Also the Euclidean distance travelled cannot cross the interior of the barriers. The main steps of the method is put forward. The preliminary results is presented based on benchmark problems.

Keywords: Heuristic, continuous location-allocation problem

Introduction

In this study, we are given a set of users, located at n fixed points, with their respective demands. We are required to locate M facilities in continuous space to serve these n users, and to find the allocation of these users to these M facilities. The objective is to minimize the sum of transportation costs. This continuous location-allocation problem can be formulated as follows:

Minimize $\sum_{i=1}^{M}\sum_{j=1}^{n} x_{ij} d(X_i, a_j)$

(1)

Subject to

$$\sum_{i=1}^{M} x_{ij} = w_j, \quad j = 1, \ldots, n$$

(2)

$$X_i = (X_i^1, X_i^2) \in S \subset \Re^2, \quad i=1,\ldots, M$$

(3) $x_{ij} \geq 0, \quad i = 1, \ldots M; \quad j = 1,\ldots, n$

(4)

where M is the number of facilities to be located, S is the feasible region to be considered, x_{ij} is the quantity assigned from facility i to customer j, $i = 1, \ldots$

M; j = 1, ..., n, $d(X_i, a_j)$ is the Euclidean distance between facility i and customer j, $a_j = (a_j^1, a_j^2) \in \Re^2$ is the location of customer j, $X_i = (X_i^1, X_i^2)$ are coordinates of facility i, w_j is the demand, or weight, of customer j.

The objective function (1) is to minimize the sum of the transportation costs. Constraints (2) guarantee that the total demand of each customer is satisfied. Constraints (3) describe the restricted regions and Constraints (4) refer to the non-negativity of the decision variables.

There is however a shortage of papers on the facility location problem with barriers. This problem is the constrained Weber problem which is also known as the Weber problem in the presence of forbidden regions and/or barriers to travel. This was initially investigated by Katz and Cooper (1981). They considered a Weber problem with the Euclidean metric and with one circular barrier. A heuristic algorithm was suggested that is based on a sequential unconstrained minimization technique for nonlinear programming problems. Hansen et al. (1982) provided an algorithm for solving the location problem when the set of feasible points is the union of a finite number of convex polygons. Other studies include Aneja and Parlar (1994) and Butt and Cavalier (1996) who developed heuristics for the median problem with *lp* distance and barriers that are closed polyhedra. Batta, Ghose, and Palekar (1989) obtained discretization results for median problems with l_1-distance and arbitrarily shaped barriers by transforming these problems into equivalent network location problems. Their results were generalized by Hamacher and Klamroth (2000) for arbitrary block norms, although it is not possible to transform these problems to the analogous network location problems. Bischoff and Klamroth (2007) proposed a genetic algorithm based solution to the problem..

Our aim is to introduce a constructive heuristic as an efficient method to solve this typical problem of facility location-allocation.

The Barriers

Proposition : For any P, X, and a barrier, segment PX must intersect two walls $W(P)$ and $W(X)$ to be invisible.

Proof : Assume that P and X not in the boundary of the barrier. Suppose that line segment PX touches one point at the edges of the barrier. Since the barrier is convex polygonal, the point must be a vertex of the barrier. So, P and X is visible, and there is no wall. Suppose that line PX intersects the edges of the barrier at point A and B. Since the barrier is convex, the line segment $AB \subset PX$ must lie inside the barrier and A lies at one edge of the barrier and B at another edge of it. Thus, PX intersects two walls.

Facing walls or faces are the wall on which the segment PX intersects. Consider Figure 3. Suppose that PX intersects $T_5 T_6$ at A and $T_{10} T_{11}$ at B. If line segment $PA < PB$, then P is facing $T_5 T_6$ and X is facing $T_{10} T_{11}$, else if line segment $PA > PB$, then P is facing $T_{10} T_{11}$ and X is facing $T_5 T_6$.

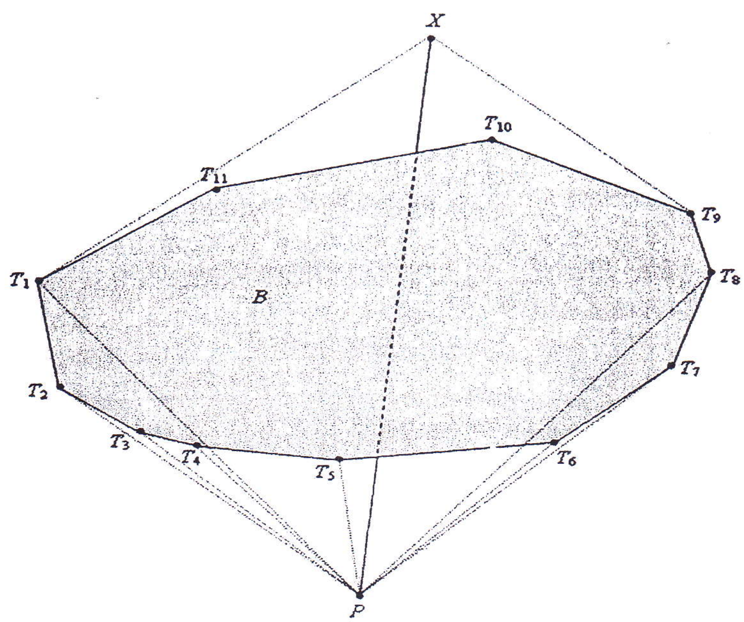

Figure 3

Finding The Last Visible Vertices Of The Barrier With *P*

Do backward method is based on Figure 3. In general, let the wall be $T_k T_{k+1}$, from above example based on Figure 3, here $k = 5$. Check if PT_{k-1} is obstructed by $T_k T_{k+1}$; if not, check if PT_{k-2} is obstructed by $T_{k-1} T_k$ or $T_k T_{k+1}$; if not, and so on check if PT_{k-i} , $i \in I$, is obstructed by $T_{k-(i-1)} T_{k-(i-2)}$, ..., or $T_k T_{k+1}$. Once the obstacle found, stop. The promising path is PT_{k-i+1}

Do forward method is based on Figure 3. In general, let the wall be $T_k T_{k+1}$, from above example based on Figure 3, $k = 5$. Check if PT_{k+2} is obstructed by

$T_k T_{k+1}$; if not, check if PT_{k+3} is obstructed by $T_k T_{k+1}$ or $T_{k+1} T_{k+2}$; if not, check if PT_{k+1+i}, $i \in I$, is obstructed by $T_k T_{k+1}$, ..., or $T_{k+i-1} T_{k+i}$. Once the obstacle found, stop. The promising path is PT_{k+i}.

From "*do backward*" and "*do forward*", points T_2 and T_7 are the two promising vertices of the barrier where one can travel from P. We do the same method from X. One can see that XT_1 and XT_9 are the promising paths. The shortest path from P to X ($\tilde{d}(P,X)$) is obtained by checking:

$$\tilde{d}(P,X) = \min\{[d(P,T_2)+d(T_2,T_1)+d(T_1,X)],\ [d(P,T_7)+d(T_7,T_8)+d(T_8,T_9)+d(T_9,X)]\}$$

Method of Checking The Intersection Point

Let $T_{k-1}(x_{k-1}, y_{k-1})$, $T_k(x_k, y_k)$, and $T_{k+1}(x_{k+1}, y_{k+1})$ be three vertices of the barrier and $P(a,b)$ be the customer's point. To check using the backward method whether PT_{k-1} intersects $T_k T_{k+1}$, first find the intersection point. Any point lying in line segment $T_k T_{k+1}$ in term of λ-function; that is $f(\lambda)$ and $g(\lambda)$ are given by $f(\lambda) = (x_k - x_{k+1})\lambda + x_{k+1}$ and $g(\lambda) = (y_k - y_{k+1})\lambda + y_{k+1}$ where $\lambda \in (0, 1)$.

Suppose that the intersection point is $I(\gamma_1, \gamma_2)$. Then we have $f(\lambda) = (x_k - x_{k+1})\lambda + x_{k+1} = \gamma_1$ or $\lambda = \frac{\gamma_1 - x_{k+1}}{x_k - x_{k+1}}$. If $\lambda > 1$ or $\lambda < 0$, the intersection point $I(\gamma_1, \gamma_2)$ is out of line segment $T_k T_{k+1}$, otherwise $I(\gamma_1, \gamma_2) \in T_k T_{k+1}$ and hence PT_{k-1} intersects $T_k T_{k+1}$.

Preliminary Result

Consider Figure 4. Suppose that we want to place a facility to serve four users, P_1, P_2, P_3, and P_4 whose coordinate locations are known, in such a way that the total Euclidean distance or total transportation cost from the facility to the users can be minimized.

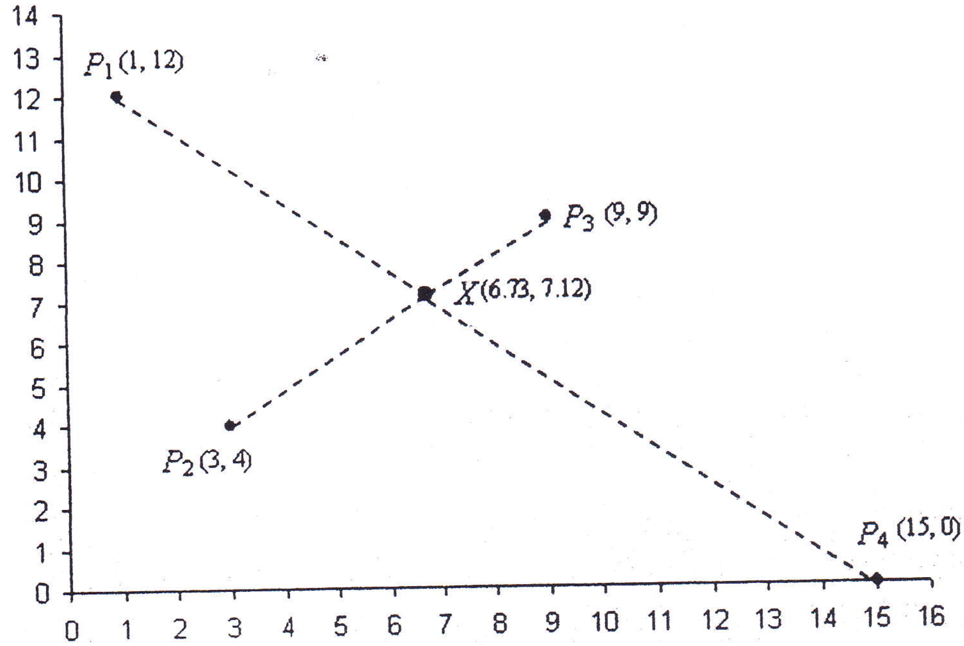

Figure 4

Choosing any point in the convex hull of the users' points as initial facility location, and then applying Cooper's iterative method (Cooper, 1964 and Gamal and Salhi, 2001) we obtain an optimal location for the facility at point (6.73 , 7.12). The optimal solution is a global one due to the convexity properties of the objective function (Cooper, 1964).

Now consider Figure 5 where there exists two convex polygonal barriers, B_1 and B_2. We cannot travel along the interior of the barriers but we can travel through their vertices or along their boundaries (Aneja and Parlar, 1994).

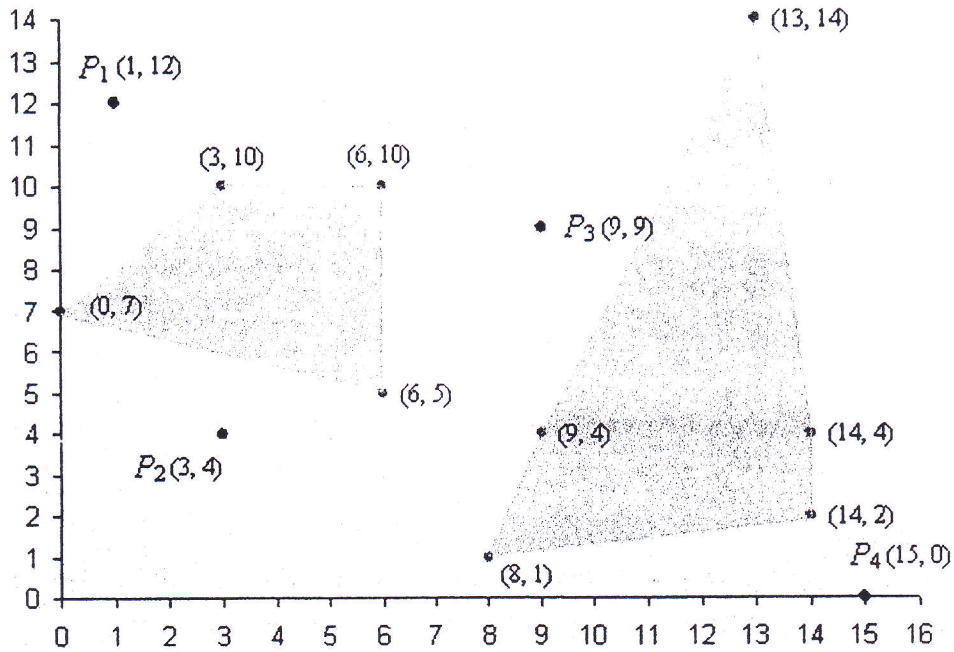

Figure 5

The single facility problem with the presence of barriers can be formulated as:

$$\text{Minimize } \sum_{j=1}^{n} w_j \, d_B(\mathbf{X}, \mathbf{a}_j)$$

$$\text{Subject to} \quad \mathbf{X} = (X^1, X^2) \in \mathrm{S} \subset \Re^2$$

where S is the feasible region to be considered and $d_B(\mathbf{X}, \mathbf{a}_j)$ is defined as the length of a shortest feasible path connecting the facility and user j and not intersecting the interior of the barriers. In contrast to the classical facility location problem (1), the facility location problem with barriers is in general non-convex (Bischoff and Klamroth, 2007).

The dotted line in Figure 6 is the minimum total distance from user j, $j = 1, 2, 3, 4$ to the facility whose location is at point (6.73 , 7.12) found earlier. But this facility location is no longer optimal because the objective function is not convex.

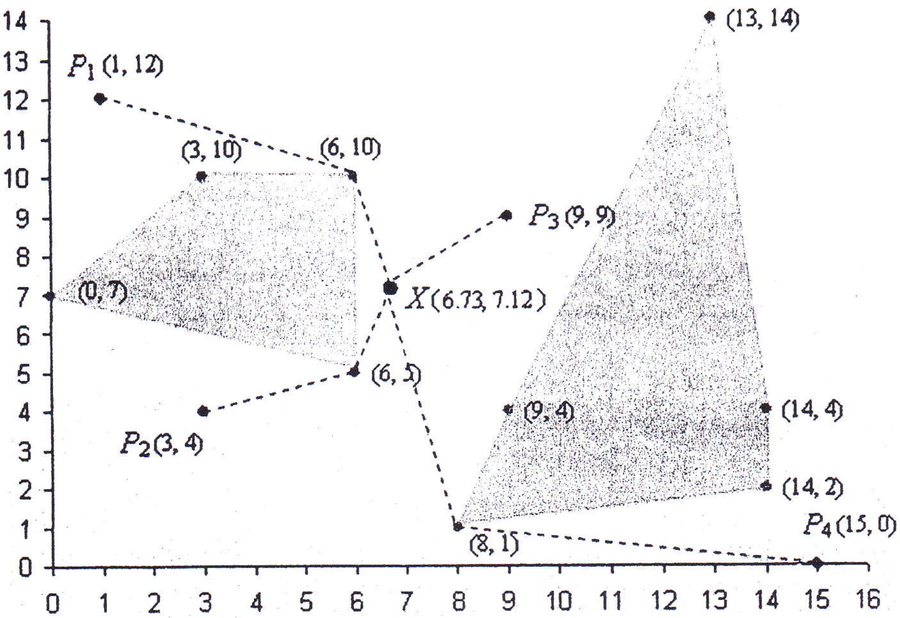

Figure 6

One way to improve the location of the facility is to consider its location with respect to three vertices of the barriers and one user's point as shown in Figure 7.

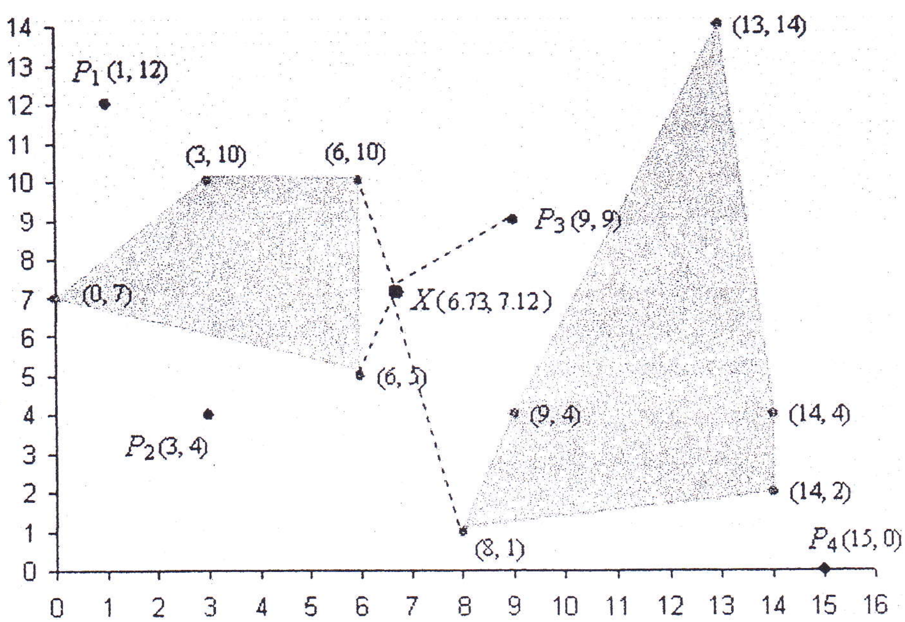

Figure 7

Using (6.73 , 7.12) as initial facility location, and then applying Cooper's iterative method, the facility location shifts to point (6.87 , 6.18) as shown in Figure 8. This new location is considered to be the best location for this small problem example.

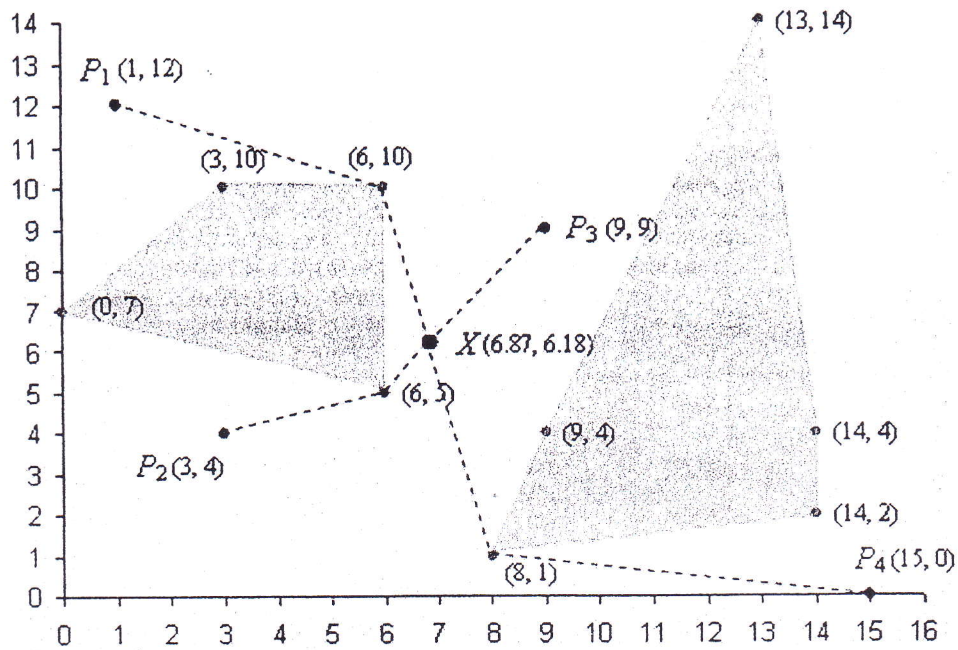

Figure 8

FUTURE WORK

The outline of the method has been put forward. It is going to be developed further. This method will be implemented to solve the facility location-allocation problem. Algorithm developed will be tested on some benchmark problems and the results will compared with the ones found so far in the literatures.

REFERENCES

Aneja, P. Y. and Parlar, M., 1994. Algorithms for Weber Facility Location in the Presence of Forbidden Regions and/or Barriers to Travel. *Transportation Science* **28**, 70-76.

Batta, R., Ghose, A. and Palekar, U., 1989. Locating facilities on the manhattan metric with arbitrarily shaped barriers and convex forbidden regions. *Transportation Science* **28** (1), 70-76.

Bischoff, M. and Klamroth, K., 2007. An Efficient Solution Method for Weber Problem with Barriers Based on Genetic Algorithms. *European Journal of Operational Research* **177**, 22-41.

Butt, S. E. and Cavalier, T. M., 1996. An Efficient Algorithm for Facility Location in the Presence of Forbidden Regions. *European Journal of Operational Research* **90**, 56-70.

Cooper, L. 1964. Heuristic Methods for Location-Allocation Problems. *SIAM Review* **6**, 37-53.

Gamal, M. D. H. and Salhi, S., 2001. Constructive Heuristics for the Uncapacitated Location-Allocation Problem. *Journal of the Operational Research Society* **51**, 1233-1240.

Hamacher, H. and Klamroth, K., 2000. Planar Location Problems with Barriers under Polyhedral Gauges. *Annals of Operations Research* **96**, 191-208.

Hansen, P., Peeters, D. and Thisse, J. F., 1982. An Algorithm for a Constrained Weber Problem. *Management Science* **28**, 1285-1295.

Katz, I. N. and Cooper, L., 1981. Facility Location in the Presence of Forbidden Regions, I: Formulation and the Case of Euclidean Distance with One Forbidden Circle. *European Journal of Operational Research* **6**, 166-173.

Salhi, S. and Gamal, M. D. H., 2003. A GA Based Heuristic for the Uncapacitated Continuous Location-Allocation Problem. *Annals of Operations Research* **123**, 203-222.