A predictor-corrector regula falsi type for solving a nonlinear equation of a single variable *

M. Imran

mimran@unri.ac.id

Laboratorium Matematika Terapan, Jurusan Matematika
Fakultas Matematika dan Ilmu Pengetahuan Alam, Universitas Riau
Pekanbaru (28293), Indonesia

Abstract

A modified regula falsi method, for solving a nonlinear equation of a single variable, based on applying another linear interpolation as corrector without any addition of function evaluations is presented in this paper. The method is known as predictor corrector method. The method has been tested in series of published examples. The numerical results show that the new method is very effective.

Keywords: *Pegasus method, Illinois method, regula falsi method, enclosing root*

1 Introduction

We consider an iterative method for computing approximate solutions of a nonlinear equation

$$f(x) = 0, \tag{1}$$

where f is a real value continuous function on $[a, b] \subset \mathbb{R}$. Suppose $f(x) = 0$ has a simple root in $[a, b]$. Classical regula falsi method (see [1]) find a simple root of the nonlinear equation (1) by repeated linear interpolation between the current breaking interval estimates. This method is attractive since convergence to a root of (1) is guaranteed. There is a distinct short coming, however, one endpoint is retained step after step, whenever a concave or convex region of $f(x)$ has been reached. Several modified regula falsi methods had been introduced to overcome these weaknesses. In [2], Dowell and Jarratt showed that a modification of this method, Illinois method, was superior to several other common methods (bisection, regula falsi, regula falsi + bisection). In [3], the same authors described an alternative modification, the Pegasus method, with superior asymptotic convergence properties. Anderson and Björk [4] used a quadratic interpolation in stead of a scaling linear interpolation to improve the regula falsi method. Naghipoor at. al. [5] used a predictor corrector strategy to improve the regula falsi method, using the regula falsi method as a predictor and theirs proposed method as a corrector. In this paper we give another modification of the regula falsi based on applying another linear interpolation as a corrector and incorporating the Pegasus method strategy. In the third section we give the numerical examples in which we compare the proposed method with the regula falsi, Illinois and Pegasus methods in the same precision.

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2 Proposed Method

Suppose that $[a, b]$ is an initial interval that the nonlinear equation (1) has a simple root, α, in it. We apply the classical regula falsi method by applying a linear interpolation from $(a, f(a))$ to $(b, f(b))$, that is

$$\frac{y - f(a)}{x - a} = \frac{f(b) - f(a)}{b - a} \tag{2}$$

and setting $y = 0$ and $x = c$ to obtain

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}. \tag{3}$$

Then we apply the other linear interpolation between $(a, f(b))$ and $(b, f(a))$, that is

$$\frac{y - f(b)}{x - a} = \frac{f(a) - f(b)}{b - a} \tag{4}$$

and setting $y = 0$ and $x = d$, we obtain

$$d = \frac{bf(b) - af(a)}{f(b) - f(a)} \tag{5}$$

Now we have three of subintervals, see Figure 1, with three cases.
Case 1: If $c < d$, we have three subcases. If $f(a)f(c) < 0$ then b is replaced with c. If $f(d)f(b) < 0$ then a is replaced with d. Otherwise a is replaced with c and set $f(b) = \frac{f(a)f(b)}{f(a)+f(c)}$.
Case 2: If $c > d$ we have three subcases. If $f(a)f(d) < 0$ then b is replaced with d. If $f(b)f(c) < 0$ then a is replaced with c. Otherwise b is replaced with c and set $f(a) = \frac{f(a)f(b)}{f(b)+f(c)}$.
Case 3: If $c = d$, we have two subcases. If $f(a)f(c) < 0$ then b is replaced with c and set $f(a) = \frac{f(a)f(b)}{f(b)+f(c)}$. If $f(b)f(c) < 0$ then a is replaced with c and set $f(b) = \frac{f(a)f(b)}{f(a)+f(c)}$.

The algorithm: Modified regula falsi method
Step 1: For a given a, b (initial interval) and ϵ (tolerance).
Step 2: Compute

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}, \qquad d = \frac{bf(b) - af(a)}{f(b) - f(a)}.$$

Step 3: If $c < d$
    if $f(a)f(c) < 0$ set $b = c$, and $x_i = c$
    else if $f(b)f(d) < 0$ set $a = d$ and $x_i = d$
    else set $a = c$, $f(b) = \frac{f(a)f(b)}{f(a)+f(c)}$ and $x_i = c$.
  else if $c > d$
    if $f(a)f(d) < 0$, set $b = d$ and $x_i = d$
    else if $f(b)f(c) < 0$, set $a = c$ and $x_i = c$
    else set $b = c$, $f(a) = \frac{f(a)f(b)}{f(b)+f(c)}$ and $x_i = c$.
  else if $c = d$
    if $f(a)f(c) < 0$, set $b = c$, $f(a) = \frac{f(a)f(b)}{f(b)+f(c)}$ and $x_i = c$
    else if $f(b)f(c) < 0$, set $a = c$, $f(b) = \frac{f(a)f(b)}{f(a)+f(c)}$ and $x_i = c$
Step 4: if $|x_{i+1} - x_i| < \epsilon|x_{i+1}|$ or $|f(x_{i+1})| < \epsilon$ then stop.
Step 5: Set $i = i + 1$ and go to step 2.

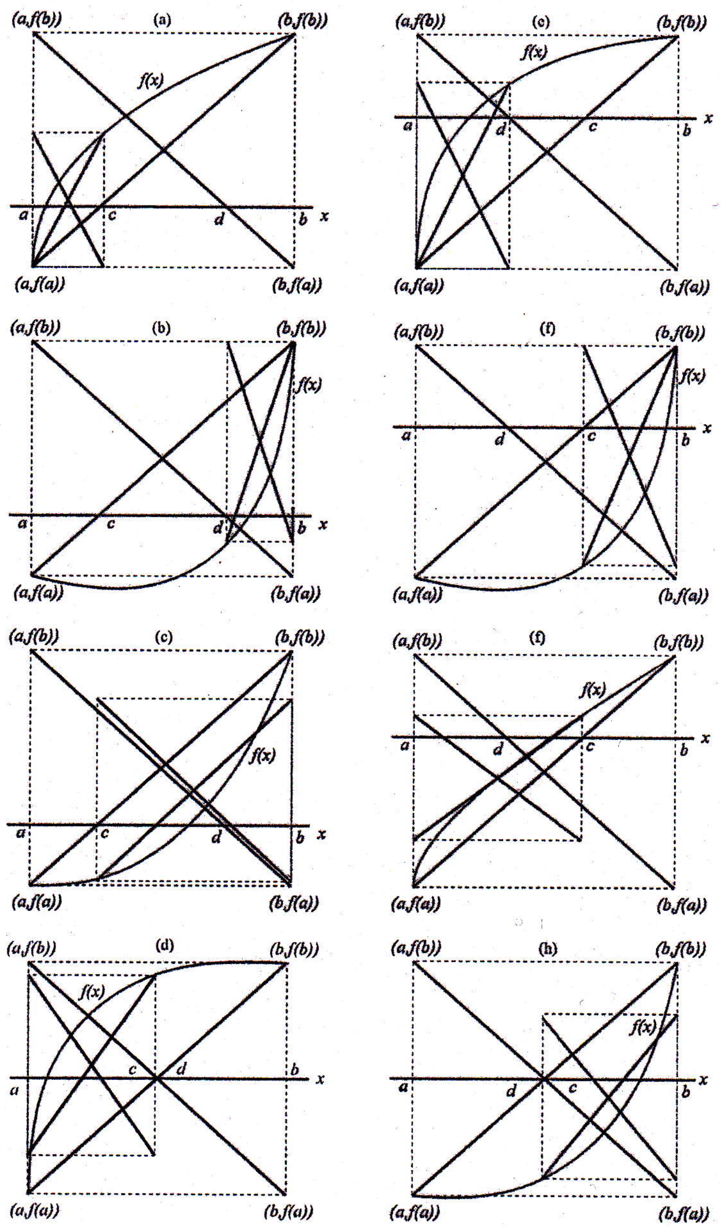

Figure 1: Geometric illustration different cases of the improved regula falsi method

3 Numerical Simulations

In this section we will consider the numerical examples and see the number of iterations that are required for the given accuracy. In all simulation recorded here ϵ was 1×10^{-12} and the maximum number of iterations n was 200. The following stopping criteria is used for our computer program:

(i) $|x_{i+1} - x_i| < \epsilon |x_{i+1}|$ or

(ii) $|f(x_{i+1})| < \epsilon$

As we see in the Table 1 that the new modified regula falsi method is effective and comparable to the regula falsi, Illinois and Pegasus methods.

Table 1: Comparisons of the number of iterations of the old methods and proposed method

No.	$f(x)$	Initial intervals		The Number of Iterations				roots
		a	b	RFM	IM	PM	IPM	
1.	$11x^{11} - 1$	0.1	0.9	39	11	9	8	0.8041330975036
		0.01	1	93	14	13	8	
		0.05	1.5	+200	22	19	18	
2.	$x^3 - 2x - 5$	2	3	26	7	7	6	2.0945514815423
		1	3.5	39	9	8	8	
		1.5	2.8	24	8	7	6	
3.	$xe^{x^2} - \sin^2(x)$ $+3\cos(x) + 5$	-1.8	-0.5	80	11	8	8	-1.2076478271309
		-2	0	161	12	10	10	
		-2	1	166	13	11	11	
4.	$e^x - 1 + \frac{1}{2}x^2$	-1	2	109	12	11	10	0.0000000000000
		-1	2.5	154	13	11	10	
5.	$e^x - 5x^2$	0	1	23	8	7	6	0.6052671213146
		0.1	1.5	35	9	8	7	
		0.1	0.9	20	8	7	6	
6.	$xe^x - 1$	-1	1	25	8	8	7	0.5671432904098
		0	1	24	8	7	6	
		0.05	0.9	20	8	7	6	
7.	$e^{x^2+7x-30} - 1$	2.8	3.1	39	11	8	7	3.0000000000000
		2.6	3.2	115	13	12	11	
		2	3.5	+200	21	20	18	
8.	$x\sin(x) + 1$	0	2	6	5	5	5	1.1141571408719
		0.001	1.8	6	6	5	5	
9.	$\frac{1}{x} - \sin(x) + 1$	-1.3	-0.5	17	8	6	6	-0.6294464840733
		-1.5	-0.05	+200	13	11	10	
10.	$\frac{1}{x} + \log(x) - 100$	0.005	0.03	39	10	7	8	0.0095556044376
		0.001	0.05	+200	13	12	10	
		0.0001	0.1	+200	17	14	13	

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Kampus Unand Limau Manis, Padang-25163 Telp. (0751) 73224, 71671
Fax. (0751) 73118 Email : icmsa2009_unand@yahoo.co.id

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Nama	:	M. Imran
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Benar-benar telah menyampaikan makalah penelitian pada acara tersebut di Balai Sidang Bung Hatta The Hills Hotel Bukittinggi pada tanggal 9 s/d 10 Juni 2009.

Demikianlah surat keterangan ini dibuat untuk dapat dipergunakan sebagaimana mestinya.

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