A MODIFICATION OF UJEVIC METHOD FREE FROM DERIVATIVE

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ABSTRACT
In this paper we discuss a modification of Ujevic method, by approximating a derivative in the method with a central difference, for solving a nonlinear equation. We show that the order of convergence of the proposed method is three. We verify the theoretical results on relevant numerical problems and compare the behavior of the proposed method with Ujevic method.
***Key words**: Ujevic method, Newton method, Third-order method.*

ABSTRAK
Pada kertas kerja ini didiskusikan modifikasi metode Ujevic, dengan mengaproksimasi turunan yang muncul di metode Ujevic menggunakan beda pusat, untuk menyelesaikan persamaan nonlinear. Secara analitik ditunjukkan bahwa meode baru ini berorde three. Hasil kajian toritis didukung oleh komputasi numerik melalui beberapa fungsi yang dipilih. Metode hasil modifikasi dibandingkan juga dengan metode Ujevic.
***Kata kunci**: metode Ujevic, metode Newton, metode berode tiga*

INTRODUCTION
Finding a numerical method to obtain approximation roots of a nonlinear equation is an active reseach in numerical analysis. Recently, researchers have developed several methods to approximate a simple root of the nonlinear equations. One of the developed methods is two step method that combines weighted Newton method with the method derived using a quadrature rule proposed by Ujevic [5]. This method can be written as

$$y_n = x_n - \alpha \frac{f(x_n)}{f'(x_n)} \qquad (1)$$

$$x_{n+1} = x_n + 4(x_n - y_n)\frac{f(x_n)}{3f(x_n) - 2f(z_n)} \qquad \text{.....(2)}$$

Ujevic [5] shows that the best choice for α is 0.5. Thander and Mandal [4] improved Ujevic method by replacing the weighted Newton method with Halley method. They do not show the order of convergence of their method analytically. Ahmad et al. [1] avoid the derivative appearing in denominator (1) by replacing weighted Newton method with method proposed by Wu and Fu [6], they end up with the following iterative method

$$y_n = x_n - \frac{\frac{1}{2}f^2(x_n)}{pf^2(x_n) + f(x_n) - f(x_n - f(x_n))} \qquad (3)$$

$$x_{n+1} = x_n + 4(x_n - y_n)\frac{f(x_n)}{3f(x_n) - 2f(z_n)} \qquad \text{.....(4)}$$

where $p \in \mathbf{R}$. They show that the iterative method (3) and (4) is of order two.

PROPOSED METHOD
Now we suggest a new iterative method by approximating the derivative in (1), by central difference, that is

$$f'(x_n) \approx \frac{f(x_n + f(x_n)) - f(x_n - f(x_n))}{2f(x_n)}. \qquad (5)$$

Substituting (5) into (1) and modifying the coefficient in (2), we arrive in the following iterative method

$$y_n = x_n - \alpha \frac{2f^2(x_n)}{f(x_n + f(x_n)) - f(x_n - f(x_n))} \quad (6)$$

$$x_{n+1} = x_n + 2(x_n - y_n)\frac{f(x_n)}{3f(x_n) - 4f(z_n)}. \quad .....(7)$$

In the following theorem we show that if $\alpha = 0.5$ then the iterative method (6) and (7) is of order three.

Theorem 1 Let $f : D \subseteq \mathbf{R} \rightarrow \mathbf{R}$ for an open interval *D*. Assume that *f* has continuously first, second and third derivative in the interval *D* and *f* has a simple root $\alpha \in D$. If x_0 is sufficiently close to α, then the method defined by (6) and (7) satisfies the following error equation:

$$e_{n+1} = \left(-\frac{F_3}{12F_1} - \frac{F_1F_3}{6} + \frac{F_2^2}{4F_1^3} \right) e_n^3 + O\left(e_n^4\right),$$

where $F_j = f^{(j)}(\alpha)$, for *j*=1,2,3.

NUMERICAL EXAMPLES

Now, we employ the modification of Ujevic method (MUM), Eqs.(6) and (7) to solve some nonlinear equations and compare it with Ujevic method (UM). All numerical computations have been carried out in a Maple 13 environment with Intel Core I5-2450 CPU-2.50 GHz based PC. The following test problems have been used with stopping criterion $|f(x_{n+1})| < 1 \times 10^{-16}$ or $|x_{n+1} - x_n| < 1 \times 10^{-16}$ or the maximum number of iteration=100.

We use the following test functions taken from [3] and also used in [2] that are

$$f_1(x) = x^3 - 2x - 5, \qquad \alpha = 2.0945514815423265 91.$$

$$f_2(x) = x(x-3) - 4\sin^2(x), \qquad \alpha = 3.019612470173171086.$$

$$f_3(x) = \sin(x) - 0.5, \qquad \alpha = 0.523598775598298873.$$

$$f_4(x) = x\exp(-x), \qquad \alpha = 0.0.$$

Table 1. Comparisons of the modified Ujevic method and Ujevic method

$f(x)$	x_0	*n*		NFE		COC	
		UM	MUM	UM	MUM	UM	MUM
f_1	1.95	4	4	12	16	2.00	3.00
	2.05	4	3	12	12	2.00	3.00
	2.15	4	3	12	12	2.00	3.00
f_2	2.00	5	5	15	20	2.00	3.00
	2.90	4	3	12	12	2.00	3.00
	3.20	5	3	15	12	2.00	3.00
f_3	1.54	5*	4	15	16	2.00	3.00
	0.70	4	3	12	12	2.00	3.00
	-1.00	6	5	18	20	2.00	3.00
f_4	-0.50	6	4	18	16	2.00	3.00
	0.25	5	3	15	12	2.00	3.00
	0.75	6	5	18	20	2.00	3.00

Displayed in Table 1 are initial guesses (x_0), the number of iterations (*n*), the number of function evaluations (NFE) and the computational order of convergence (COC). The star sign (*) in the number of iterations indicates that the method converges to a different root of the nonlinear equation. We see in the Table 1 that, over all the modified Ujevic method need less iteration to obtain the approximation root of the nonlinear equations considered. The analytic result of order convergence is in agreement with the computational experiments.

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